Kinetic Energy
A Survey

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$$E_t = \frac{mv^2}{2g_c 7000}$$

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The intent of this survey is to acquaint the reader with a working concept of kinetic energy. The survey dives deeply into kinetic energy and covers what it is, how it came to be and does it apply to small arms projectiles.

There are a bunch of numbers and variables. I know that sounds dry and dismal, but it's the only way to convey the origins of kinetic energy. Don't worry, I do all of the math step by step. I explain each steep and make commentary along the way.

The Primary Definitions

Force is any influence when put upon an object at rest or in motion will change the motion of that object and can change its velocity and or direction. Force is given by:

$$F = ma$$

Kinetic Energy (KE or E_k) is the energy possess by a ridged body that does not deform or change shape and is not a rotating body (**E_r**). This is the **classic statement** of "*half mass times the square if its velocity.*" There are many forms of kinetic energy. Here is the classic statement:

$$KE = \frac{1}{2} mv^2$$

Translational Kinetic Energy (E_t) is the **specific statement** for kinetic energy as given by a particular acceleration of gravity. This is the translational kinetic energy equation:

$$E_t = \frac{mv^2}{2g_c}$$

Work is the capacity to transfer energy to or from an object by force.

*For those purists reading this survey. I understand that Einstein's theory of relativity for gravity (space time) has supplanted Newton's theory of gravity. However, Newton's theory of gravity within most celestial body still holds true.

A History of Kinetic Energy

1665 Sir Isaac Newton did his great work. On **July 5th 1686**, with the help of Edmond Halley, Newton publishes his work; the *Philosophiae Naturalis Principia Mathematica*. Newton purposed the Second Law of Motion. Newton's Second Axiom is stated as: *"The change of motion is proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed."*

This second law is expressed as:

F = ma

1676-1689 Gottfried Leibniz and Johann Bernoulli described energy as a living force (*Vis viva*). Both are accredited with the development of $\mathbf{mv^2}$. During this time Leibniz promoted what we now call Conservation of Energy ($\mathbf{mv^2}$). Newton disagreed with Leibniz and promoted the Conservation of Momentum (**mv**).

As a personal note, I believe Newton most certainly knew of $\mathbf{v^2}$. **a** from **F = ma** is acceleration and acceleration can be expressed as $\mathbf{v^2}$.

1722, Willem Jakob 's Gravesande published his experiment of dropping brass balls from different heights into sheets of clay. He determined that the penetration depth was proportional to the square of velocity. 's Gravesande found that a ball with twice the speed of another would leave an impact creator four times as deep.

1740, the Marquise du Châtelet Gabrielle-Émilie le Tonnelier de Breteuil used 's Gravesande's data to mathematically prove that energy is exponential not proportional to the square of the velocity. du Châtelet's work now sets $\mathbf{mv^2}$ apart from **mv** mathematically**.**

1741, Daniel Bernoulli publishes and article showing the coefficient of **½** *(½ mvv).*

1802, Thomas Young first uses the term *energy* in a lecture for the Royal Society. This is where energy is separated from force; Vis viva. [Kinetic] energy is stated as:

$$\mathbf{E = \frac{1}{2}mv^2}$$

1829, Gaspard-Gustave de Coriolis publishes a book that gives [kinetic] energy its application to mechanical work.

1849, William Thomson "Sir Lord Kelvin" is credited with the term "kinetic energy". Lord Kelvin, working alongside William John Macquorn Rankine recognized that *through motion there is active work.*

1853, Rankine is credited for the term "potential energy". Potential Energy is stated as:

U = mgh

September 27th 1905, Albert Einstein published his original work containing this equation for energy:

$$E=\frac{m\,c^2}{\sqrt{1-\frac{y^2}{c^2}}} \qquad (28)$$

This equation is more commonly known as:

$$\mathbf{E = mc^2}$$

Einstein wrote, *"Energy cannot be created or destroyed; it can only be changed from one form to another."*

The Dimensional Constant

First, let me introduce you to the basic variables used in this survey. All dimensions, terms and units of measure (UOM) are in Imperial units from the English Engineering System.

Whereas:

F is the pound force ($\mathbf{lb_f}$)
m is the pound mass in pounds avoirdupois ($\mathbf{lb_m}$)
d is distance in feet (**ft**)
t is time in seconds (**s**)
v is the velocity in feet per second (**d/t**)
g is acceleration of gravity in feet per second squared ($\mathbf{d/t^2}$)
$\mathbf{g_c}$ is the dimensional constant $\frac{\mathbf{md}}{\mathbf{Ft^2}}$

We as hunters and shooters uses the translational kinetic energy equation. It's the dimensional constant that gives the *acceleration of gravity* a UOM. For example, the acceleration of gravity for the Moon is $\mathbf{5.315ft/s^2}$. Therefore, the dimensional constant on the moon for the acceleration of gravity is $\mathbf{5.315md/Ft^2}$ or **5.315**. For Mars the acceleration of gravity is $\mathbf{12.2375ft/s^2}$ and the dimensional constant is $\mathbf{12.2375md/Ft^2}$ or **12.2375**. Here on earth the acceleration of gravity is $\mathbf{32.1739ft/s^2}$ and the dimensional constant is $\mathbf{32.1739md/Ft^2}$ or **32.1739**. But we as hunters and shooters use a more specific acceleration of gravity. It's called the *local acceleration of gravity*. The local acceleration of gravity is $\mathbf{32.163ft/s^2}$. Therefore, the dimensional constant we use is $\mathbf{32.163md/Ft^2}$ or **32.163**. Here is the translational kinetic energy equation that we hunters and shooter use:

$$\mathbf{E_t = \frac{mv^2}{2 \cdot 32.163}}$$

or

$$\mathbf{E_t = \frac{mv^2}{2 \cdot 32.163 \cdot 7000}}$$

The **7000** sets the equation equal to pounds when using grains as the bullet mass.

The Kinetic Energy Equation

Getting right to the point, kinetic energy is a measurement of work. Some folks believe that kinetic energy is the work of lifting a weight or the torque of a torque wrench (the cross product). Kinetic energy is neither weight nor the torque of a torque wrench. Kinetic energy is the capacity to do work due to the motion of an object. Movement is the principal factor.

When calculating kinetic energy, a datum line needs to be established. The datum line for a bullet is nothing more than the energy at the muzzle or along specified liner points to impact. Usual the points are range increments of 100yds. But the measurements can be at any increments you desire.

Let's set up the kinetic energy equation as it relates to a falling object. The object starts at rest and falls through a datum line. Kinetic energy is given by:

$$\mathbf{KE} = \mathbf{wz}$$

Whereas:
KE is kinetic energy
w is the weight of the body
z is the average velocity of the falling body.

These are the terms for **w**.

$$\mathbf{w} = \frac{\mathbf{m \cdot g}}{\mathbf{g_c}}$$

These are the terms for **z**.

$$\mathbf{z} = \frac{\mathbf{v_1 + v_2 \cdot t}}{\mathbf{2}}$$

At this time, I am going put the factored terms of **wz** together. I will also put all the variables over one divisor bar and add the dot product between each variable. Typically, this is not done, but for the purpose of this survey, it makes it easier to visualize the equations. I will supplant $\mathbf{v_1}$ and $\mathbf{v_2}$ as **v. v** is a representation of the average velocity from v_1 and $\mathbf{v_2}$. I will also add a **1** over top of the **2** to represent the averaging of **v**.

$$\mathbf{KE} = \frac{\mathbf{m \cdot g \cdot 1 \cdot v \cdot t}}{\mathbf{g_c \quad 2}} =$$

Now, you will notice the ½ appears in the middle of the equation and not in front of the mass. Again, this is because it represents the averaging of velocities v_1 and v_2 from variable **z**. In pointing this out, this is the number one question many people have about the kinetic energy equation. Why is only half of the mass used in the kinetic energy equation? The answer of course is <u>all</u> the mass is used. Placing the numeral ahead of the variable is normal mathematical practice. Besides it's easier to say "half mass times velocity squared" rather than "mass times velocity square divided by two."

$$\mathbf{KE} = \frac{m \cdot g \cdot \mathbf{1} \cdot v \cdot t}{g_c \quad \mathbf{2}} =$$

We will now remove the one half. This will not affect the equation as we are doing a mathematical exercise.

$$\mathbf{KE} = \frac{m \cdot g \cdot v \cdot t}{g_c} =$$

From this form I will factor the equation to its limits, then go through the equation until it is reduced to the final two variables.

Let us factor **g** and $\mathbf{g_c}$.

$$\mathbf{KE} = \frac{m \cdot \mathbf{g} \cdot v \cdot t}{\mathbf{g_c}} =$$

g and $\mathbf{g_c}$ factor too.

$$\mathbf{KE} = \frac{m \cdot \frac{\mathbf{d}}{\mathbf{t^2}} \cdot v \cdot t}{\frac{\mathbf{m \cdot d}}{\mathbf{F \cdot t^2}}} =$$

Now I will factor **v.**

$$\mathbf{KE} = \frac{m \cdot \frac{d}{t^2} \cdot \mathbf{v} \cdot t}{\frac{m \cdot d}{F \cdot t^2}} =$$

v factors too;

$$\mathbf{KE} = \frac{m \cdot \frac{d}{t^2} \cdot \mathbf{d} \cdot t}{\frac{m \cdot d}{F \cdot t^2} \cdot \mathbf{t}} =$$

Okay it's time to rearrange that nasty and complex fraction in the middle of the equation. The upper fraction is the UOM that make up the local acceleration of gravity (**g**) and the lower fraction represents the dimensional constant ($\mathbf{g_c}$).

$$\mathbf{KE} = \frac{m \cdot \frac{\mathbf{d}}{\mathbf{t^2}} \cdot d \cdot t}{\frac{\mathbf{m \cdot d}}{\mathbf{F \cdot t^2}} \cdot t} =$$

To rearrange this complex fraction (step 1) the top fraction in the numerator will be *divided by* the bottom fraction in the denominator. Since we don't like to *divide* fractions *by* fractions, we'll multiplying the top fraction by the reciprocal of the bottom fraction (step 2).

1. $$\mathbf{KE} = \frac{m \cdot \mathbf{d} \div \mathbf{m} \cdot \mathbf{d} \cdot d \cdot t}{\mathbf{t^2} \quad \mathbf{F} \cdot \mathbf{t^2} \cdot t} =$$

2. $$\mathbf{KE} = \frac{m \cdot \mathbf{d} \cdot \mathbf{F} \cdot \mathbf{t^2} \cdot d \cdot t}{\mathbf{t^2} \cdot \mathbf{m} \cdot \mathbf{d} \cdot t} =$$

At this point we are factored as far as we can.

$$\mathbf{KE} = \frac{m \cdot d \cdot F \cdot t^2 \cdot d \cdot t}{t^2 \cdot m \cdot d \cdot t} =$$

Okay, time to cross-cancel. We have **t²** and **t** in the numerator and **t²** and **t** in the denominator. The **t²** and **t** above the divisor bar will cancel out the **t²** and **t** below the divisor bar.

$$\mathbf{KE} = \frac{m \cdot d \cdot F \cdot \mathbf{t^2} \cdot d \cdot \mathbf{t}}{\mathbf{t^2} \cdot m \cdot d \cdot \mathbf{t}} =$$

$$\mathbf{KE} = \frac{m \cdot d \cdot F \cdot \cancel{\mathbf{t^2}} \cdot d \cdot \cancel{\mathbf{t}}}{\cancel{\mathbf{t^2}} \cdot m \cdot d \cdot \cancel{\mathbf{t}}} =$$

$$\mathbf{KE} = \frac{m \cdot d \cdot F \cdot d}{m \cdot d} =$$

Now, we will cross-cancel the **m** and **d** in the numerator and **m** and **d** in the denominator.

$$\mathbf{KE} = \frac{\mathbf{m} \cdot d \cdot F \cdot \mathbf{d}}{\mathbf{m} \cdot \mathbf{d}} =$$

$$\mathbf{KE} = \frac{\cancel{\mathbf{m}} \cdot d \cdot F \cdot \cancel{\mathbf{d}}}{\cancel{\mathbf{m}} \cdot \cancel{\mathbf{d}}} =$$

$$\mathbf{KE} = d \cdot F =$$

And now you know where *foot-pound force* comes from. It's time to remove the dot product from between the **d** and **F.**

$$\mathbf{KE} = \mathbf{d} \cdot \mathbf{F} =$$

$$\mathbf{KE} = \mathbf{dF}$$

Since **distance** is expressed as the foot and **force** is expressed as pound force, this is our answer:

$$\mathbf{KE} = \mathbf{dF}$$

or

$$\mathbf{KE} = \mathbf{ft\text{-}lb_f}$$

or

Kinetic Energy *equals* foot-pound force

We have taken the fully reduced kinetic energy equation and factored it. Then reduce it to an equation that expresses kinetic energy. As you can see with all the different variables that represent time, distance, force and mass, we were able to factor the equation down to the two the last two variables.

So, there you go. There is no magic to the kinetic energy equation. It’s just a matter of understanding that foot-pound force is the measurement of work. Specifically, that of objects in motion.

Kinetic Energy to Translational Kinetic Energy

Now that we understand what and where foot-pound force comes from. It's time to understand the difference between kinetic energy and translational kinetic energy.

In this the second part we will delineate the point where **KE** equation becomes the $\mathbf{E_t}$ equation. We'll start with **w** and **z** factored:

$$\mathbf{KE} = \frac{\mathbf{m} \cdot \mathbf{g}}{\mathbf{g_c}} \cdot \frac{\mathbf{1}}{\mathbf{2}} \cdot \mathbf{v} \cdot \mathbf{t} =$$

The **1** is removed as 1 *times* any value is still that value.

$$\mathbf{KE} = \frac{\mathbf{m} \cdot \mathbf{g} \cdot \mathbf{v} \cdot \mathbf{t}}{\mathbf{g_c} \cdot \mathbf{2}} =$$

As you can see, we have left the numerical value of **2** in the denominator. I did this because we are moving from a mathematical concept to a numerical concept. Therefore, this equation needs real numeric values.

We will now combine the **g** and **t**.

$$\mathbf{KE} = \frac{m \cdot \mathbf{g} \cdot v \cdot \mathbf{t}}{g_c \cdot 2} =$$

$$\mathbf{KE} = \frac{m \cdot \mathbf{g} \cdot \mathbf{t} \cdot v}{g_c \cdot 2} =$$

Now it just so happens that **g** *times* **t** is **v**.

$$\mathbf{KE} = \frac{m \cdot \mathbf{g} \cdot \mathbf{t} \cdot v}{g_c \cdot 2} =$$

$$\mathbf{KE} = \frac{m \cdot \mathbf{v} \cdot v}{g_c \cdot 2} =$$

Now we have **v** *times* **v** and they are stated as $\mathbf{v^2}$. This is a question some people also have. Why is the velocity used twice? The **v** squared is a derivative of Newton's Second Law; $\mathbf{v^2}$ is *equal* to **a**. It also comes out by the factoring of the terms: **gt** *equals* **v** and **v** *times* **v** *equals* $\mathbf{v^2}$**.**

$$\mathbf{KE} = \frac{m \cdot \mathbf{v} \cdot \mathbf{v}}{g_c \cdot 2} =$$

$$\mathbf{KE} = \frac{m \cdot \mathbf{v^2}}{g_c \cdot 2} =$$

Now as you can see we have just created the first recognizable part of the "kinetic energy equation" and translational kinetic energy equations with $\mathbf{v^2}$ visible.

$$\mathbf{KE} = \frac{m \cdot \mathbf{v^2}}{g_c \cdot 2} =$$

We are going to remove the dot product and move the variables next to each other, as is normal in mathematical operations.

$$\mathbf{KE} = \frac{m \cdot v^2}{g_c \cdot 2} =$$

$$\mathbf{KE} = \frac{mv^2}{g_c 2} =$$

The next step is to move the numerical value of **2** to the front of the variable, as is fundamentally done in mathematics. This is where the KE equation becomes the E_t equation.

$$\mathbf{KE} = \frac{mv^2}{g_c\mathbf{2}} =$$

$$\mathbf{E_t} = \frac{mv^2}{\mathbf{2}g_c} =$$

Since we as hunters and shooter use grains (**gr**) as our UOM for the weight, I will now add in **7000** to set the equation equal to pounds.

$$\mathbf{E_t} = \frac{mv^2}{2g_c} =$$

$$\mathbf{E_t} = \frac{mv^2}{2g_c\mathbf{7000}} =$$

Now, all that would be needed is to change the variable for $\mathbf{g_c}$ to its numerical value which is the local acceleration of gravity of **32.163**. So, I will reintroduce the dot product back into the equation to delineate the numerical values.

$$\mathbf{E_t} = \frac{mv^2}{2\mathbf{g_c}7000} =$$

$$\mathbf{E_t} = \frac{mv^2}{2 \cdot \mathbf{32.163} \cdot 7000} =$$

$$\mathbf{E_t} = \frac{mv^2}{2 \cdot 32.163 \cdot 7000}$$

There it is! This is the equation used to develop real translational kinetic energy values. Particularly the ones in the back of reloading manuals or on the packaging of your favorite small arms manufacturer's ammunition.

Translation Kinetic Energy to Kinetic Energy

Okay it's now time for the last part. I will use the E_t equation from above and go back to the general statement of **KE.**

First, I would remove the seven thousand, because were not concerned with grains as a value of mass.

$$E_t = \frac{mv^2}{2 \cdot 32.163 \cdot \mathbf{7000}} =$$

$$E_t = \frac{mv^2}{2 \cdot 32.163} =$$

Second, I would remove the **32.163**. I can do this, because the dimensional constant is only there to give the equation UOM.

$$E_t = \frac{mv^2}{2 \cdot \mathbf{32.163}} =$$

$$E_t = \frac{\mathbf{mv^2}}{\mathbf{2}} =$$

Third, let's put the **2** to the front of the equation and put a **1** over top as is normally done in mathematics.

$$E_t = \frac{mv^2}{\mathbf{2}} =$$

$$E_t = \frac{\mathbf{1}}{\mathbf{2}} \cdot mv^2 =$$

Here is where the equation goes from E_t to **KE**.

$$E_t = \frac{1}{2} \cdot mv^2 =$$

$$\mathbf{KE} = \frac{\mathbf{1}}{\mathbf{2}} \cdot \mathbf{mv^2} =$$

Now I'm going to remove the dot product.

$$\mathbf{KE} = \frac{\mathbf{1}}{\mathbf{2}} \cdot \mathbf{mv^2} =$$

$$\mathbf{KE} = \frac{\mathbf{1}}{\mathbf{2}} \mathbf{mv^2} =$$

There it is, the old classic statement; *Kinetic energy equals half mass times the square of its velocity.*

Applying Translational Kinetic Energy?

One way to look at translational kinetic energy values is as raw data. For example, a range card is created. The translational kinetic energy output may be a column of values. The range might be in increments of 100yds each. At this point you have raw data. The question now is how to process it. This is the dilemma most hunters and shooters have.

Common sense tells us the high the translational kinetic energy value the greater the energy. But what is the right amount of energy. There is really no list of applicable foot-pound force values for game animals. I have read many, many times that “1000 foot pounds” is right for deer. Also, a “ton of energy” for elk and “3200 foot pounds” for brown bear. But other than that, it’s all a guess.

The truth is, translational kinetic energy alone will not tell you how much energy is needed. There needs to be more information. That information is the construction of the bullet. Later in this survey I propose a way to evaluate a bullet-cartridge combination using bullet construction.

Rotational Kinetic Energy

I thought I might add rotational kinetic energy (**E_r**) into the mix. The reason? It's because some bullet manufacturers tout a bullets ability to cut or auger at impact of a game animal or ballistics medium. You will see that the energy yielded is very small. This is why I reject the notion of auguring a medium or for the taking of a game animal as unlikely.

Okay, at this point you must be half asleep with all math. So, I'm going to write the formula for rotational kinetic energy and run quickly through the equation.

Rotational kinetic energy:

$$E_r = \frac{xm \cdot \left(\frac{CAL}{2 \cdot 12}\right)^2 \cdot \left[2 \cdot 3.14159 \cdot \left(\frac{V \cdot 12}{TW}\right)\right]^2}{2 \cdot 32.163 \cdot 7000}$$

Whereas:
E_r is the Rotational Kinetic Energy.
x is the conversion in a percentage of a projectile's Center of Gravity due to shape as solid cylinder or sphere
m is the mass of the bullet in grains.
CAL is the bullet diameter to 3 places in inches.
2 is to obtain the bullet radius.
12 is the conversion factor to set the equation equal to the foot.
2 *times* **3.14159** is the coefficient to the revolution/s (radians) for the rotational velocity of a bullet.
v is the velocity of the bullet in feet per second.
TW is the bullet twist rate in inches.
12 is the conversion factor ti set the equation equal to the foot.
2 is the average velocity from the original equation of **wz** for a falling object.
32.163 is the dimensional constant **g_c**.
7000 is the conversion factor to set the equation equal to the pounds.
Again, here is the rotational kinetic energy equation:

$$E_r = \frac{x \cdot m \cdot \left(\frac{CAL}{2 \cdot 12}\right)^2 \cdot \left[2 \cdot 3.14159 \cdot \left(\frac{v \cdot 12}{TW}\right)\right]^2}{2 \cdot 32.163 \cdot 7000}$$

Now let's plug in some numbers from our Standard bullet of .**30cal**, **180gr** with a muzzle velocity of **2700fps**. We will also use the standard twist rate of **1** twist in **10** inches and **45.5** % of a bullet's center of gravity.

$$E_r = \frac{.455 \cdot 180 \cdot \left(\frac{.308}{2 \cdot 12}\right)^2 \cdot \left[2 \cdot 3.14159 \cdot \left(\frac{2700 \cdot 12}{10}\right)\right]^2}{2 \cdot 32.163 \cdot 7000}$$

Okay, the two fraction with parenthesizes are:

.**30 cal** *divided by* **2** *times* **12** is **.01283**

2700 *times* **12** *divide by* **10** is **3240**

$$E_r = \frac{.455 \cdot 180 \cdot \left(\frac{.308}{2 \cdot 12}\right)^2 \cdot \left[2 \cdot 3.14159 \cdot \left(\frac{2700 \cdot 12}{10}\right)\right]^2}{2 \cdot 32.163 \cdot 7000}$$

$$E_r = \frac{.455 \cdot 180 \cdot \left(.01283\right)^2 \cdot \left[2 \cdot 3.14159 \cdot \left(3240\right)\right]^2}{2 \cdot 32.163 \cdot 7000}$$

Now we will square **.01283** which is **.0001646**

$$E_r = \frac{.455 \cdot 180 \cdot \left(.01283\right)^2 \cdot \left[2 \cdot 3.14159 \cdot 3240\right]^2}{2 \cdot 32.163 \cdot 7000}$$

$$E_r = \frac{.455 \cdot 180 \cdot .0001646 \cdot \left[2 \cdot 3.14159 \cdot 3240\right]^2}{2 \cdot 32.163 \cdot 7000}$$

Next is to calculate the numbers with in the brackets which are **20357.5**

$$E_r = \frac{.455 \cdot 180 \cdot .0001646 \cdot \left[2 \cdot 3.14159 \cdot 3240\right]^2}{2 \cdot 32.163 \cdot 7000}$$

$$E_r = \frac{.455 \cdot 180 \cdot .0001646 \cdot \left[20357.5\right]^2}{2 \cdot 32.163 \cdot 7000}$$

Now we square the number within the bracket and that is **414427806.25**

$$E_r = \frac{.455 \cdot 180 \cdot .0001646 \cdot \left[\mathbf{20357.5}\right]^2}{2 \cdot 32.163 \cdot 7000}$$

$$E_r = \frac{.455 \cdot 180 \cdot .0001646 \cdot \mathbf{414427806.25}}{2 \cdot 32.163 \cdot 7000}$$

So now we multiply through the numerator and denominator.

$$E_r = \frac{\mathbf{.455 \cdot 180 \cdot .0001646 \cdot 414427806.25}}{\mathbf{2 \cdot 32.163 \cdot 7000}}$$

$$E_r = \frac{\mathbf{5596976}}{\mathbf{450282}}$$

And finally, the numerator *divided by* the denominator to get our answer.

$$E_r = \frac{\mathbf{5596976}}{\mathbf{450282}}$$

E_r = **12.43** or
12.43ft-lb_f

So, our answer is a poultry **12.43** foot-pound force of rotational kinetic energy.

Well... how much do I have to say about this. As you can see there is very little rotational energy created by the spin of a bullet.

Let me say this, “rotational kinetic energy does not cause tissue damage upon impact.” In fact, I'll go one step further and say this, “rotational kinetic energy does not cause bullet blow-up due to thin jackets.” I believe the blowing up of jackets is caused by the stress from the rifling. Thus, weakening the jacket its self. Then the bullet exits the muzzle. The force of the atmosphere blows the jacket off and the bullet fails within a few yards of the muzzle.

Estimated Effective Energy

This next section may or may not be interesting to you. It is about Estimated Effective Energy (**EEE)** EEE is an umbrella term for formula that calculated knock down or killing power. EEE is purely imperial. Some formula use $\mathbf{E_t}$ as a base, others use momentum.

I have done some analysis of several EEE in the past. I used the drawings of Dr. Martin L. Fackler Col USA (ret.). The drawings are based on temporary and permeant wound channel of a ballistics gelation impacts. The sampling was: .22 long rifle with a 40gr bullet, 5.56mm cartridge with a 55gr bullet, .30-30 Winchester with a 150gr bullet, 12ga shot shell with a 437gr slug and .308 Winchester with a 150gr bullet. I believe the sampling was too small to make a definitive conclusion as to whether any of the EEE I tested were valid.

There are maybe 20 such formula that I have run across in my 40 some years of hunting and shooting. Some of them work quit well for some people and their shooting and/or hunting experiences. For some EEE does nothing. So, these folks are left with translational kinetic energy.

I can say this, even translational kinetic energy has its limitations. Translational kinetic energy is limited to one bullet wight with no regard for bullet construction. But it is possible to calculate the energy of each differing bullet at impact. Here is an example of differing $\mathbf{E_t}$ using a **180gr** bullet with a muzzle velocity of **2700** at impact.

Bullet	bc	v at Impact	E_t
Solid Copper Hollow Point (SCHP)	**.453**	**2501**	**2500**
Partition	**.474**	**2510**	**2518**
Pointed soft point	**.483**	**2513**	**2525**
Boat Tail Core Bonded	**.507**	**2522**	**2542**
Hollow Point Boat Tail (VLD)	**.576**	**2543**	**2584**

As you can see every bullet's output is proportionate. So, you can pick **bc** (ballistic coefficient), **v** at impact or $\mathbf{E_t}$**.** All values will tell you the same thing. But this is a big but. The construction of the bullet is not account for. Further down the text you will see how the construction of a bullet differs from $\mathbf{E_t}$

Impact Penetration Factor

I'll spend some time explaining **IPF** and how to used it. I provide charts in the reference section.

Let’s start with the equation for sectional density. The equation for sectional density is expressed as:

$$\mathbf{sd} = \frac{\mathbf{w}}{\mathbf{d^2 \cdot 7000}}$$

Whereas:
sd is the sectional density of the bullet.
w is weight of the bullet in grains.
d is the diameter of the bullet squared.
7000 sets the equation correct to pounds.

What sectional density conveys is how much weight a bullet applies per each square of its diameter at the bullet base. The greater the weight of a specific caliber bullet the greater the section density. This is scientifically true. Sectional density is the “**P**” in **IPF.**

Here is the **I** in **IPF**; translational kinetic energy.

$$\mathbf{E_t} = \frac{\mathbf{mv^2}}{\mathbf{2 \cdot 32.163 \cdot 7000}}$$

The **F** in **IPF** is the value yielded.

So now I'm going to show you how this works. I will use the standard bullet and cartridge. The standard bullet is a **180gr .30cal** and is a pointed soft point. The standard cartridge is a **Springfield .30-06'**. The muzzle velocity is **2700fps** and standard design function (**id**) is **1.00**.

I'm going to move quickly though the math. By now you should be able the follow along at a faster pace, so let’s crunch some numbers.

Again, here is the translational kinetic energy.

$$\mathbf{E_t} = \frac{\mathbf{180 \cdot 2700^2}}{\mathbf{2 \cdot 32.163 \cdot 7000}}$$

The numerator is:
180 *times* $\mathbf{2700^2}$

$\mathbf{2700^2}$ is **7290000**

180 *times* **7290000** is **1312200000**

The denominator is:
2 *times* **32.163** *times* **7000**

2 *time* **32.163** is **64.326**

64.326 *times* **7000** is **450282**

Now the numerator *divided by* the denominator:
1312200000 *divided by* **450282** is **2914.177**

or

$\mathbf{2914ft\text{-}lb_f}$

The sectional density is a **180gr** bullet is **.271.** The design function (**id)** is **1.00**

$$\mathbf{IPF = \frac{2914 \cdot .271}{1.00}}$$

The numerator is:
2914 *times* **.271** is **789.694**

Now the numerator *divided by* the denominator is:
789.694 *divided by* **1.00** is **789.694**

or

IPF = **790**

Here is how IPF works. Again, I will use the standard bullet-cartridge combination. The bullets I am comparing are all **180gr**. They are as follows:
Pointed soft point
Partition
Solid Copper Hollow Point (SCHP)
Core Bonded
Hollow Point Boat Tail (VDL)

All of these bullets have the same sectional density; **.271** and weight; **180gr**. But we know that each of these bullets behave differently upon impact. Each of these bullets must have a design function to state the differing behaviors.

Here is the different **id** at impact with a muzzle velocity of **2700fps**:

Bullet	id
Pointed soft point	**1.00**
Hollow Point Boat Tail (VLD)	**1.00**
Core Bonded	**.800**
Partition	**.833**
Solid Copper Hollow Point (SCHP)	**.769**

So here are the different values as dictated by **IPF**:

The Pointed Soft Point.

$$IPF = \frac{2525 \cdot .271}{1.00}$$

IPF = 684

Hollow Point Boat Tail (VLD)

$$IPF = \frac{2584 \cdot .271}{1.00}$$

IPF = 700

Partition

$$IPF = \frac{2510 \cdot .271}{.833}$$

IPF = 820

Core Bonded

$$IPF = \frac{2542 \cdot .271}{.800}$$

IPF = 863

Solid Copper Hollow Point (SCHP)

$$IPF = \frac{2500 \cdot .271}{.769}$$

IPF = 881

Here is a comparison between the differing bullets

Bullet	E_t	IPF
Solid Copper Hollow Point (SCHP)	**2500**	**881**
Partition	**2518**	**820**
Pointed soft point	**2525**	**684**
Boat Tail Core Bonded	**2542**	**863**
Hollow Point Boat Tail (VLD)	**2584**	**700**

You can see the there is an **84ft-lb$_f$** difference in translational kinetic energy for the bullets lowest to highest. There is only a **3%** difference between them. Not too much of a difference. This would suggest you could pick any of these bullets and expect a similar outcome. For **IPF** there is a **197**-point difference from highest to lowest. That's a **22%** difference. I think the IPF bares out the true ability of each bullet and what you can expect from them. I would also say, a higher IPF may not indicated a better bullet for the intended purpose. The bullet with the lowest IPF may be all that is needed. So now you can see how your favorite bullet-cartridge can stack up against other bullet-cartridge combinations.

A Few Last Thoughts

Over the last 40 years I have subscribed to one or more shooting magazine. In this time, I have read a dozen or so articles written about kinetic energy. Most of them are misleading at best and some just down right wrong. The last straw for me was an aerospace engineer that wrote an article about kinetic energy. His supposition was that bullets melt holes in metal. The test medium was a rear leaf spring from a car. He was just down-right wrong. The so-called heat ring around the hole was nothing more than a stress mark left behind by the bullet punching a hole through the leaf spring; not melting it…

So, there you go… Good hunting and shooting my friends.

Thank you,

Greg Glover

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Abbreviations

Whereas:
E_t is translational kinetic energy ($mv^2/2g_c$) in foot-pound force.

Tke is Translational kinetic energy.
pwc is permanent wound channel volume in cubic inched.
twc is temporary wound channel volume in cubic inches.
M is Momentum in pound force per second.
IPF is Impact Penetration Factor (as a value only).
TKO is Taylor knock-out value.
bw is the bullet weight.
bc is ballistic coefficient.
cal is the diameter of the bullet.
sd is sectional density in pounds per cross section squared.
Pen. is penetration.
ft/s is feet per second.
$ft\text{-}lb_f$ is foot-pound force.
gr is grains.
in is the inch.
in^2 is inches squared.
in^3 is inches cubed.
s is second.
lb is pound.
F is force.
m is mass.
d is distance.
t is time.
mv is mass times velocity.
v is d/t

*Some used in this survey.

References

EEE Formulas

Impact Penetration Factor: $\frac{\mathbf{Tke} \cdot \mathbf{sd}}{\mathbf{id}}$

Original IPF: $\mathbf{Tke} \cdot \mathbf{sd}$

John Wootter's Lethality Index: $\mathbf{Tke} \cdot \mathbf{cal} \cdot \mathbf{sd}$

Shock Power Index: $\mathbf{Tke} \cdot \mathbf{cal}$

A Square Penetration Index: $\frac{\mathbf{Tke} \cdot \mathbf{sd}}{\mathbf{100} \cdot \mathbf{cal}}$

Yielding Point: $\frac{\mathbf{Tke}}{\mathbf{cal}}$

Momentum Equation: $\mathbf{bw} \cdot \mathbf{v}$

Momentum Equation (Set to Pound Feet): $\frac{\mathbf{bw} \cdot \mathbf{v}}{\mathbf{7000}}$

Taylor Knockout Value: $\frac{\mathbf{bw} \cdot \mathbf{v} \cdot \mathbf{cal}}{\mathbf{7000}}$

Momentum Theory: $\frac{\mathbf{bw} \cdot \mathbf{v}}{\mathbf{10000}}$

A Square Relative Performance Index: $\frac{\mathbf{.001} \cdot \mathbf{bw} \cdot \mathbf{v}^2 \cdot \mathbf{cal}}{\mathbf{7000}}$

Optimum Game Weight: $\mathbf{v}^3 \cdot \mathbf{bw}^2 \cdot \mathbf{1.5} \cdot \mathbf{.000000000001}$

Hornady HITS: $\frac{\mathbf{bw}^2 \cdot \mathbf{v}}{\mathbf{700000} \cdot \mathbf{cal}^2}$

Kinetic Pulse: $\mathbf{Tke} \cdot \frac{\mathbf{bw} \cdot \mathbf{v}}{\mathbf{7000}\, \mathbf{g_c}}$

Design Function (**id**.) for Bullets

Handgun Type Projectiles
id at Impact Velocity
heat treated cast lead 0 to 900fps **.300**
heat treated cast lead 900 to 1100fps **.333**
heat treated cast lead 1100 to 1400fps **.370**
heat treated cast lead 1400 to 2100fps **.400**

military ball round 0 to 2100fps **1.11**

cast lead 0 to 600fps **.270**
cast lead 600 to 1100fps **.333**
cast lead 1100 to 1400fps **.300**
cast lead 1400 to 1800fps **.333**

jacketed round nose 0 to 600fps **.250**
jacketed round nose 600 to 1100fps **.270**
jacketed round nose 1100 to 1400fps **.300**
jacketed round nose 1400 to 2100fps **.333**

jacketed flat point 0 to 600fps **.250**
jacketed flat point 600 to 1100fps **.270**
jacketed flat point 1100 to 1400fps **.300**
jacketed flat point 1400 to 2100fps **.333**

jacketed hollow point 0 to 600fps **.250**
jacketed hollow point 600 to 1100fps **.270**
jacketed hollow point 1100 to 1400fps **.333**
jacketed hollow point 1400 to 1700fps .**300**
jacketed hollow point 1700 to 2100fps .**333**

jacketed solid 0 to 1100fps **.270**
jacketed solid 1100 to 1400fps **.300**
jacketed solid 1400 to 2100fps **.333**

Shotgun & Black Powder Type Projectiles
id at Impact Velocity
shotgun slug 0 to 600fps **.400**
shotgun slug 600 to 1100fps **.455**
shotgun slug 1100 to 1400fps **.769**
shotgun slug 1400 to 1800fps **.952**

sabot 0 to 600fps **.370**
sabot 600 to 1100fps **.500**
sabot 1100 to 1400fps **.555**
sabot. 1400 to 1800fps **.769**

(Minni ball type)
cast lead 0 to 600fps **.400**
cast lead 600 to 1100fps **.455**
cast lead 1100 to 1400fps **.769**
cast lead 1400 to 1800fps **.952**

(of wheel weight type)
cast lead 0 to 600fps **.400**
cast lead 600 to 1100fps **.455**
cast lead 1100 to 1400fps **.714**
cast lead 1400 to 1800fps **.870**

<u>id at Impact Velocity</u>
(of round ball type)
shot 0 to 600fps **.434**
shot 600 to 1100fps **.625**
shot 1100 to 1400fps **.769**
shot 1400 to 1800fps **.952**

<u>Rifles Type Projectiles</u>
<u>id at Impact Velocity</u>
hollow point 0 to 4500fps **1.18**
(of varmint type)

polycarbonate tip 0 to 4500fps **1.18**
(of varmint type)

military ball round 0 to 3000fps **1.11**

hollow point VLD 0 to 1100fps **.555**
hollow point VLD 1100 to 1400fps **.769**
hollow point VLD 1400 to 2000fps **.833**
hollow point VLD 2000 to 2900fps **1.00**

cast lead 0 to 600fps **.400**
cast lead 600 to1100 fps **.455**
cast lead 1100 to 1400fps **.769**
cast lead 1400 to 2100 fps **.952**

(of .30-30' type)
jacketed flat point 0 to 900fps **.400**
jacketed flat point 900 to 1100fps **.625**
jacketed flat point 1100 to 1400fps **.769**
jacketed flat point 1400 to 2100fps **.910**
jacketed flat point 2100 to 2700fps **1.00**

(of .45-70 type)
jacketed hollow point 0 to 600fps **.370**
jacketed hollow point 600 to 1100fps **.455**
jacketed hollow point 1100 to 1400fps **.714**
jacketed hollow point 1400 to 2100fps **1.00**
jacketed hollow point 2100 to 2700fps **1.11**

(.030 thickness to ≤ .338)
jacketed soft point 0 to 900fps **.400**
jacketed soft point 900 to 1100fps **.555**
jacketed soft point 1100 to 1400fps **.769**
jacketed soft point 1400 to 2000fps **.833**
jacketed soft point 2000 to 2900fps **1.00 (standard model)**

<u>(of .030 thickness to ≤ .338)</u>
polycarbonate tip
jacketed soft point 0 to 900fps **.400**
jacketed soft point 900 to 1100fps **.555**
jacketed soft point 1100 to 1400fps **.769**
jacketed soft point 1400 to 2000fps **.827**
jacketed soft point 2000 to 2700fps **.970**
jacketed soft point 2700 to 2900fps **1.00**

id at Impact Velocity
(of .040 plus thick ≥ .338)
heavy jacketed soft point 0 to 1100fps **.400**
heavy jacketed soft point 1100 to 1400fps **.769**
heavy jacketed soft point 1400 to 2000fps **.714**
heavy jacketed soft point 2000 to 2900fps **.870**
heavy jacketed soft point 2900 to 3200fps **1.00**

partition 0 to 900fps **.400**
partition 900 to 1100fps **.625**
partition 1100 to 1400fps **.769**
partition 1400 to 1800fps **.714**
partition 1800 to 2900fps **.833**
partition 2900 to 3200fps **.952**

core bonded 0 to 900fps **.400**
core bonded 900 to 1100fps **.555**
core bonded 1100 to 1400fps **.741**
core bonded 1400 to 2000fps **.769**
core bonded 2000 to 2900fps **.800**
core bonded 2900 to 3500fps **.870**

.45-70 type
SCHP 0 to 600fps **.390**
SCHP 600 to 1100fps **.472**
SCHP 1100 to 1400fps **.769**
SCHP1400 to 2100fps **1.00**
SCHP2100 to 2700fps **1.05**
*Solid Copper Hollow Point

Standard rifle bullet
SCHP solid 0 to 1100fps **.400**
SCHP solid 1100 to 1400fps **.625**
SCHP solid 1400 to 1700fps **.555**
SCHP 1700 to 3000fps **.769**
SCHP 3000 to 3500fps **.800**
* Solid Copper Hollow Point

core bonded partition 0 to 1100fps **.400**
core bonded partition 1100 to 1400fps **.741**
core bonded partition 1400 to 1900fps **.714**
core bonded partition 1900 to 2900fps **.800**
core bonded partition 2900 to 3500fps **.870**

jacketed solid 0 to 1100fps **.400**
jacketed solid 1100 to 1400fps **.555**
jacketed solid 1400 to 2900fps **.769**
jacketed solid 2900 to 3200fps **.833**

homolithic solid 0 to 1100fps **.400**
homolithic solid 1100 to 1400fps **.555**
homolithic solid 1400 to 3000fps **.714**
homolithic solid 3000 to 3500fps **.800**

Note:
Maximum listed impact velocities are considered the threshold before a liquid impact occurs. Varmint type bullets are designed to break up upon impact and maximum impact velocities do not apply. Also, the maximum impact velocities do not apply to military ball rounds.

Impact Penetration Factor
Index of Minimum Cartridge

Group&Species	Factor	Basis of minimum
Varmints		
Prairie dogs	**53**	.223 Remington 50gr **HP** at 2000fps at 300yds.
Rock chucks	**53**	.223 Remington 50gr **HP** at 2000fps at 300yds.
Coyotes	**117**	.22-250 w/ 55gr **PSP** at 2475fps at 300yds.
Medium Game		
Pronghorn	**216**	.243 Winchester w/ 90gr **PSP** at 2226fps at 400yds.
Eastern Blacktail	**226**	7mm-30 Waters w/ 120gr **FP** at 2000fps at 150yds.
Deer under 150 lbs	**261**	.243Winchester w/ 90gr **PSP** at 2449fps at 300yds.
Big Game		
Mountain Goats	**318**	.240 Weatherby w/95gr **PSP** at 2562fps at 300yds.
Deer over 150 lbs	**376**	250-3000 Savage w/100gr **PSP** at 2800 at 100yds.
Bighorn Sheep	**399**	.25-06' Remington w/100gr **PSP** at 2886fps at 200yds.
American Bison	**582**	Sharps .40-90 w/370gr **LS** at 1285fps at 50yds.
Harvest Elk	**597**	.270 Winchester w/130gr **PSP** at 2925fps at 75yds
Moose	**710**	Springfield .30-06' w/180gr **PSP** at 2560fps at75yds.
Trophy Elk	**766**	.300 Winchester w/180gr **PSP** at 2660fps at 200yds.
American Dangerous Game		
Brown Bear	**1019**	.300 Winchester w/180gr **PPSP** at 2800fps at 50yds.
African Thin Skin Game		
To 100 lbs	**475**	.25-06' Rem. w/120gr **PPSP** at 2390fps at 300yds.
To 250 lbs	**702**	.270 Winchester w/150gr at **PPSP** 2510fps at 200yds.
To 500 lbs	**1004**	.270 Weatherby w/180gr at **PPSP** 2500fps at 200yds.
To 1000 lbs	**1127**	.300 Winchester w/ 200gr **PPSP** at 2650fps at 150yds.
To 2000 lbs	**1362**	.300 Weatherby w/220gr **PPSP** at 2650fps at 100yds.

Sectional Density Tables

BW	**.224**						
40	.114						
45	.128						
50	.143	**.243**	**.257**				
55	**.157**						
60	.171	.14	.129				
65	.185	.157	.141				
70	.199	.169	.151				
75	.181	.162	.154				
80	.194	.175	.164		**.277**		
85	.206	.184	.174				
90	.218	.194	.184	.264	.168	**.284**	**.308**
95	.230	.205	.195		.177		
100	**.242**	.216	.205	.205	.186	.177	.151
105	.254	.227	.215	.215	195	.186	.158
110		**.238**	.225	.225	.205	.195	.166
115				.236	.214	.204	.173
120				.246	.223.	.213	.181
125				.256	.232	.221	.188
130				.266	.242	.230	.196
135				**.277**	.251	.239	.203
140				.287	.260	.248	.211
145				.297	.269	.257	.281
150				.307	**.279**	.266	.226
155				.318	.288	**.275**	.233
160				.328	.297	.283	.241
165				.338	.306	.292	.248
170				.348	.316	.301	.256
175				.359	.325	.310	.264
180				.369	.334	.319	**.271**
185						.328	.279
190						.337	.286
195						.345	.294
200						.354	.301
205							.309
210							.316
215							.324
220							.331
225							.339
235							.353
250							.376

BW			**.357**				
110			.123				
115	**.323**		.129				
120			.134				
125	.171		.140				
130	.178	**.338**	.146				
135	.185		.151				
140	.192	.175	.157	**.358**			
145	.196	.181	.162				
150	.205	.188	.168	.167			
155	.212	.194	.174	.173			
160	.219	.200	.179	.178			
165	.226	.206		.184			
170	.233	.213		.190			
175	.240	.219		.195			
180	.246	.225		.200			
185	.253	.231		.206			
190	.260	.238		.212		**.375**	**.411**
195	.267	.244		.217			
200	**.274**	.250		.223		.203	.167
205	.281	.256		.229		.208	.173
210	.286	.263		.234		.213	.178
215	.294	.269		.240		.218	.182
220	.301	**.275**		.245		.223	.186
225	.308	.281		.251	**.366**	.229	.190
235	.322	.294		.262		.234	.195
250	.342	.312		**.279**	.267	.254	.211
270		.338		.301	**.288**	**.274**	.228
275		.344		.307	.293	.279	.233
286		.358		.319	.305	.291	.242
290		.363		.323	.309	.295	.245
300		.375		.343		.320	.254
325						.330	**.275**
350						.356	.296
375							.317
400							**.338**
410							.347

BW			**.429**				
200			.169				
205			.159				
210			.163				
215			.167				
220			.171				
225			.175				
235			.179				
250			.194				
270			.210				
275			.213				
286	**.416**	**.423**	.222	**.458**	**.475**		
290			.225				
300	.248	.240	.233	.204	.190		
325	.268	.259	.256	.221	.206		
350	.289	.279	**.272**	.238	.222		
375	.310	.299		.255	.237		
400	.330	.319		.272	.253		
410	**.338**	**.327**		.279	.260		
450	.371	.359		.306	.285	**.511**	
465	.384	.371		.317	.296		
500				**.341**	.317	.274	
550				.375	**.348**	.301	
600				.409	.380	**.338**	
650						.356	**.577**
690						.377	
700						.383	.300
707						.387	.303
750						.410	.322
800						.438	**.342**
850						.465	.365
900						.492	.386
950						.520	.408
1000						.547	.429